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# CALIBRATION OF FINITE ELEMENT MODELS OF METSOVO BRIDGE USING AMBIENT VIBRATION MEASUREMENTS

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**ABSTRACT:** A Bayesian inference method for structural model updating is used to develop high fidelity finite element (FE) models of the Metsovo bridge-foundation-soil system using modal characteristics identified from ambient vibration measurements. These models are representative of the initial structural condition of the bridge and can be further used for structural health monitoring purposes.

**KEY WORDS:** Structural dynamics, Modal estimation, Bayesian inference, Model calibration, Soil-Bridge Interaction.

## 1 INTRODUCTION

In Northern Greece the largest and most challenging Greek project of design, supervision, construction, operation, maintenance and exploitation of 680 km of the motorway, linking Europe with Turkish borders, has been constructed. This is the Egnatia Motorway project, carried out by Egnatia Odos S.A.

The evaluation of the actual dynamic characteristics and the development of high-fidelity finite element models of some major bridges over steep and deep ravines, in the west sector of the Egnatia Motorway, crossing particularly difficult geological terrain and obstacles, like the Metsovo bridge, has been attracting an increasing research effort.

Recent developments in Bayesian methodologies [1] based on ambient vibration measurements are used for estimating the modal frequencies and mode shapes of the bridge and their uncertainties. Due to the large size of the bridge, the mode shapes of the structure are assembled from a number of sensor configurations. The modal properties and their uncertainties are then integrated within Bayesian model updating formulations to calibrate the parameters of FE model as well as the associated uncertainty. These formulations require a moderate to very large number of repeated FE model analyses to be performed and can increase substantially the computational effort to excessive levels. Fast and accurate component mode synthesis (CMS) techniques, consistent with the finite element (FE) model parameterization, are integrated with Bayesian techniques to reduce efficiently and drastically the computational effort [2]. Further computational savings are achieved by adopting parallel computing algorithms such as the Transitional MCMC to efficiently distribute the computations in available multi-core CPUs [3].

In the context of this work, high performance computing and model reduction techniques are integrated within Bayesian tools that are used to calibrate the uncertainty in FE models of Metsovo Bridge within very reasonable computational time, despite the very large number of DOFs of the models.

## 2 BAYESIAN INVERSE MODELLING

### 2.1 Bayesian Model Calibration

Bayesian techniques are used for calibrating uncertainty models in structural dynamics based on vibration measurements. Let $\hat{\omega}_r$ be the estimated modal frequencies and $\underline{\hat{\phi}}_r \in R^{N_{0,r}}$ the modeshape components at $N_0$ measured DOFs, where $m$ is the number of observed modes. In this work, the modal properties are estimated from output only vibration measurements using the Bayesian modal parameter estimation method proposed in [1]. The Bayesian approach to model calibration deals with updating the values of the parameter set $\underline{\theta}$ associated with the structural model parameters and the prediction error parameters. Following the Bayes theorem, the uncertainty in the parameters given the measured data $D$ is quantified by the posterior distribution as

$$p(\underline{\theta} \mid D) = \frac{p(D \mid \underline{\theta})\ \pi(\underline{\theta})}{p(D)} \tag{1}$$

where $p(D \mid \underline{\theta})$ is the likelihood, $\pi(\underline{\theta})$ is the prior distribution of the uncertain parameters and $p(D)$ is the evidence of the finite element model. Assuming that the prediction errors for the modal frequencies and mode shapes are independent Gaussian zero-mean random variables with variance $\sigma_\omega^2$ and $\sigma_\varphi^2$, the likelihood is readily obtained by the following form

$$p(D \mid \underline{\theta}) \propto \frac{1}{\sigma_\omega^m \sigma_\phi^M} \exp\left[ -\frac{1}{2\sigma_\omega^2} J_\omega(\underline{\theta}) - \frac{1}{2\sigma_\phi^2} J_\phi(\underline{\theta}) \right] \tag{2}$$

where $J_\omega(\underline{\theta})$ and $J_\varphi(\underline{\theta})$ represent the measure of fit between the experimentally obtained modal data and the modal data predicted by the FE model. Moreover, the model prediction error parameters $\sigma_\omega$ and $\sigma_\varphi$ are considered to be unknown and are incorporated in the unknown parameter set $\underline{\theta}$.

Using the vector $\underline{a}_r(\underline{\theta}) = [\Phi_r^T(\underline{\theta})\Phi_r(\underline{\theta})]^{-1}\Phi_r^T(\underline{\theta})\hat{\underline{\phi}}_r$ to guarantee that the modal mode shapes $\underline{\varphi}_r(\underline{\theta})$ $r = 1,\dots,m$ predicted from a particular value of the parameter set $\underline{\theta}$ are closest to experimentally obtained $\hat{\underline{\phi}}_r$ and given that $\Lambda(\underline{\theta}) = \mathrm{diag}[\omega_r^2(\underline{\theta})]$, the functions $J_\omega(\underline{\theta})$ and $J_\varphi(\underline{\theta})$ are given by

$$J_\omega(\underline{\theta}) = \sum_{r=}^{m} \frac{a_r^T(\underline{\theta})\Lambda(\underline{\theta})a_r(\underline{\theta}) - \hat{\omega}_r^2 a_r^T(\underline{\theta})a_r(\underline{\theta})}{\hat{\omega}_r^2 a_r^T(\underline{\theta})a_r(\underline{\theta})}, \quad J_\phi(\underline{\theta}) = \sum_{r=}^{m} \frac{\| \Phi_r(\underline{\theta})a_r(\underline{\theta}) - \hat{\underline{\phi}}_r \|^2}{\| \hat{\underline{\phi}}_r \|^2} \tag{3}$$

## 2.2 Bayesian Computing Tools

The Bayesian tools for identifying FE models as well as performing robust prediction analyses are Laplace methods of asymptotic approximation and stochastic simulation algorithms. In this work the Transitional MCMC (TMCMC) stochastic simulation algorithm [5] is employed. High performance computing techniques are integrated within the TMCMC tool and fast and accurate component mode synthesis techniques [2] are used, consistent with the FE model parameterization, to efficiently handle large number of DOF in FE models. Further computational savings are achieved by adopting parallel computing algorithms to efficiently distribute the computations in available multi-core CPUs [3].

# 3 METSOVO BRIDGE

## 3.1 Description

The ravine bridge of Metsovo is the highest bridge of Egnatia Motorway, with the height of the taller pier M2 equal to 110m. The total length of the bridge is 537 m. The key of the central span is not in midspan due to the different heights of the superstructure at its supports to the adjacent piers (13,0m in pier M2 and 11,50 in pier M3) for redistributing mass and load in favor of the short pier M3 and thus relaxing strong structural abnormality.

The bridge has 4 spans , of length 44,78m /117,87m /235,00m/140,00m and three piers of which M1, 45m high, supports the boxbeam superstructure through pot bearings, while M2, M3 piers connect monolithically to the superstructure. The total width of the deck is 13,95m, for each carriageway. The superstructure is limited prestressed of single boxbeam section, of height varying from 4,00 m to 13,5m. Piers M2, M3 are founded on huge circular Ø12,0m rock sockets in a depth of 25m and 15m, respectively.

## 3.2 Finite Element Modelling

Detailed FE models are created using 3-dimensional tetrahedral quadratic Lagrange FEs to model the whole bridge. An extra coarse mesh is chosen to predict the lowest 10 modal frequencies and mode shapes of the bridge. The model has 97,636 FEs and 563,586 DOFs. In order to examine the contribution of soil conditions on the dynamic response of the bridge, two different classes of finite element models of the bridge are developed to predict the dynamic behavior of the bridge, a fixed base model and one that models the soil stiffness with translational springs attached in the two horizontal and the vertical directions at each base of the three piers and the two abutments.

The lowest ten modal frequencies of the left branch of Metsovo Bridge predicted by the fixed base model are presented in Table I and are compared to those predicted by the flexibly supported model. It can be seen from the results in Table I that soil contribution varies from 0.35% ($1^{st}$ Mode) to 2.47% ($10^{th}$ Mode). It is obvious that the effect of soil-structure interaction on dynamic response of Metsovo Bridge cannot be ignored. Finally, it is obvious that modal frequencies predicted by the bridge model including the contribution of soil are, as expected, lower than the ones for the fixed base bridge model.

*Table 1.* Modal Frequencies in Hz of left branch of Metsovo Bridge

| No | Identified Mode | Fixed Base FEM | FEM With Soil | % Difference |
|---|---|---|---|---|
| 1 | $1^{st}$ Transverse | 0.3179 | 0.3168 | 0.35 |
| 2 | $2^{nd}$ Transverse | 0.6219 | 0.6190 | 0.47 |
| 3 | $1^{st}$ Bending | 0.6462 | 0.6406 | 0.57 |
| 4 | $3^{rd}$ Transverse | 0.9897 | 0.9837 | 0.61 |
| 5 | $2^{nd}$ Bending | 1.1121 | 1.1045 | 0.68 |
| 6 | $4^{th}$ Transverse | 1.1734 | 1.1643 | 0.78 |
| 7 | $3^{rd}$ Bending | 1.5164 | 1.4988 | 1.16 |
| 8 | $5^{th}$ Transverse | 1.7117 | 1.7021 | 0.56 |
| 9 | $4^{th}$ Bending | 1.9341 | 1.9233 | 0.56 |
| 10 | $6^{th}$ Transverse | 2.3187 | 2.2615 | 2.47 |

# 4 MODAL IDENTIFICATION

## 4.1 Bridge Instrumentation

The acceleration time histories recorded by 5 triaxial and 3 uniaxial accelerometers paired with a 24-bit data logging system and an internal SD flash-card for data storage ,shown in *Fig.1*, are used to identify the modal properties of the bridge under normal operating conditions. The recorded responses are mainly due to road traffic, which ranged from light vehicles to heavy trucks, and environmental excitation such as wind loading which classifies this case as ambient modal identification.

Figure 1. Instrumentation

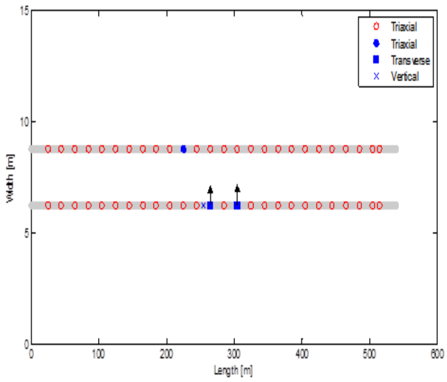


*Figure 2.* Sensor Configuration

An important aspect of this measurement system is the fact that it is wireless, since this allowed for multiple sets of repeated measurements that had to be performed for accurate mode shape identification, given the limited number of sensors and the large length of the deck. Specifically, the entire length of the deck was covered in 13 sensor configurations ,shown in *Fig.2*,with each configuration recording for 20 minutes with a sampling rate of 100 Hz. One tri-axial and three uniaxial sensors (one vertical and two horizontal), one at each side of the bridge, remained at the same position, in order to provide common measurement points along different configurations so as to enable the procedure of mode shape assembling [4].

## 4.2 Modal Identification Software

For the estimation of modal properties from the ambient acceleration data, software developed by the System Dynamics Laboratory of University of Thessaly was used. The software for Ambient (Output-only) Modal Identification has four distinct modules, namely Data insertion, Pre-processing, Modal Identification and Post processing.

The *Data insertion module* is used to load the experimentally measured acceleration time histories, even of multiple sets, into the program. The data insertion module imports any number of sensor configurations, each one represented by a MATLAB file .

In the *Pre-processing module* the user can visually inspect the Power Spectral Density (PSD) in *Fig. 3* and the Singular Value Spectrum (SVS) in *Fig. 4* of the acceleration time histories in order to obtain a rough estimate of the natural frequencies of the structure and define the frequency bands that are utilized by the Modal Identification module.

The *Modal Identification module* uses the recently proposed Bayesian methodology [1] in order to estimate the modal frequencies, mode shapes, and damping ratios. The method is based on the Fast Fourier Transform (FFT) of the acceleration signals in specific bands of interest and also provides the uncertainty in the estimates of the modal properties.

In the *Post processing module* the user can insert the geometry, define the measured degrees of freedom, assemble the full mode shapes, and perform other auxiliary actions such as animate the mode shapes in *Fig. 5*.

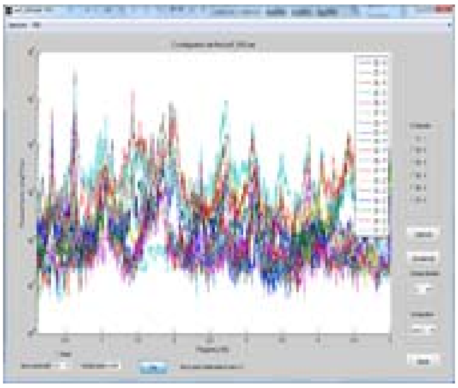

*Figure 3.* PSD Plot

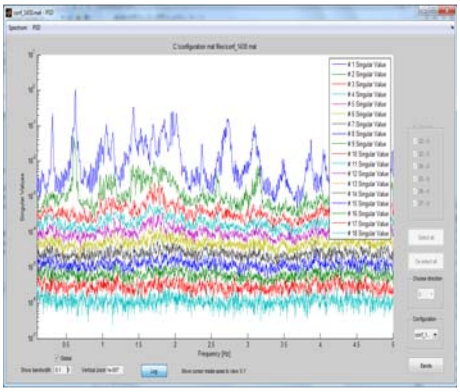

*Figure 4.* SVS Plot

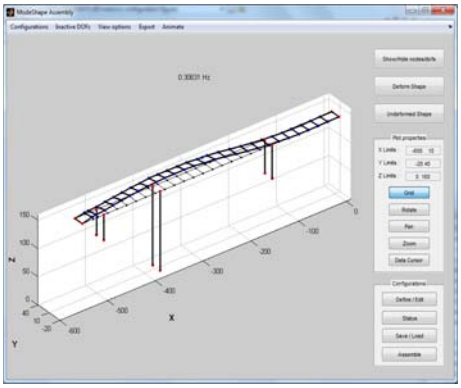

*Figure 5.* Identified Mode Shape Plot

### 4.3 Modal Identification Results

Using the software incorporating the Bayesian Methodology and the mode shape assembling algorithm, the natural frequencies and damping ratios of the structure were estimated. For comparison purposes, Table 2 presents the mean and the standard deviation (STD) of the experimentally identified modal frequencies for the lowest 10 modes of the Metsovo bridge. The experimental values should be compared to the FE model predictions in Table 1. It can be seen that the bridge is less stiff than the fixed-base FE model predicts which suggests that the FE model with soil is more appropriate to model the bridge.

*Table 2.* Experimental and model predicted natural frequencies

| No | Type of Mode | Modal Frequencies in Hz | | | | Damping Ratios (%) | |
|---|---|---|---|---|---|---|---|
| | | Experimental | | Calibrated Model | | Experimental | |
| | | Mean | STD | Mean | STD | Mean | STD |
| 1 | Transverse | 0.3063 | 0.0007 | 0.3064 | 0.0042 | 1.1 | 0.25 |
| 2 | Transverse | 0.6034 | 0.0014 | 0.5993 | 0.0058 | 1.8 | 0.24 |
| 3 | Bending | 0.6227 | 0.0008 | 0.6242 | 0.0069 | 0.85 | 0.24 |
| 4 | Transverse | 0.9646 | 0.0084 | 0.9330 | 0.0233 | 1.4 | 1.1 |
| 5 | Bending | 1.0468 | 0.0066 | 1.1107 | 0.0092 | 1.9 | 1.1 |
| 6 | Transverse | 1.1389 | 0.0049 | 1.1427 | 0.0099 | 1.1 | 0.43 |
| 7 | Bending | 1.4280 | 0.0042 | 1.4678 | 0.0140 | 1.4 | 0.64 |
| 8 | Transverse | 1.6967 | 0.0112 | 1.6549 | 0.0147 | 1.6 | 1.6 |
| 9 | Bending | 2.0053 | 0.0054 | 1.8389 | 0.0175 | 1.1 | 0.41 |
| 10 | Transverse | 2.3666 | 0.0025 | 2.2978 | 0.0252 | 0.85 | 0.16 |

## 5 CALIBRATION OF METSOVO FE MODEL

The flexibly supported FE model of Metsovo Bridge is parameterized using five parameters associated with the modulus of elasticity of one or more structural components, shown in *Fig. 6*. The model parameters scale the nominal values of the properties that they model. In order to reduce the computational effort, a parameterization-consistent component mode synthesis (CMS) technique is applied [2] resulting in a reduced model of 588 generalized coordinates, with errors in the estimates for the lowest 20 modal frequencies to be less than 0.02%. Thus, the time to solution for one run of the reduced model is of the order of a few seconds which should be compared to the 2 minutes required for solving the unreduced FE model. Also, in order to further reduce the time to solution, the computations were performed in parallel using 8 cores available from a 4-core double threaded computer.

The calibration is done using the lowest 10 modal frequencies and mode shapes identified for the structure in Table 2. The TMCMC is used to generate samples from the posterior PDF of the structural model and prediction error parameters and then the uncertainty is propagated to estimate the uncertainty in the modal frequencies of the bridge. The mean and the standard deviation of the uncertainty in the first 10 modal frequencies are presented in Table 2. It can be seen that predictions of the uncertainty for the first 10 modal frequencies are overall closer to the experimental data than the ones predicted from the nominal model in Table 1. The overall fit between the experimental and the model predicted modal frequencies is shown in *Fig. 7*.

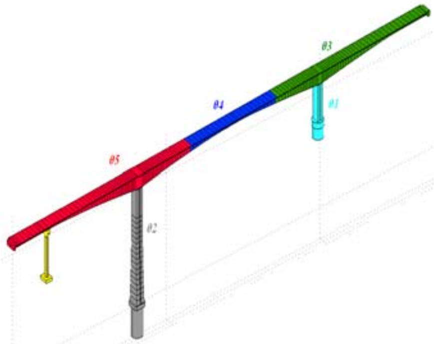

*Figure 6.* Model Parameterization

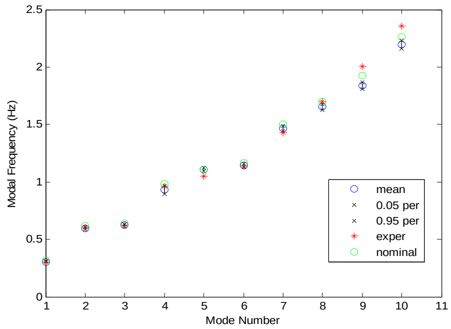


*Figure 7.* Modal Frequency Fit

## 6 CONCLUSIONS

The application of the proposed Bayesian methodology for model calibration to Metsovo Bridge demonstrated its computational efficiency and accuracy. The proposed CMS method and parallel implementation strategies allowed for two to three orders of magnitude reduction of computational time. The calibrated FE models of the Metsovo bridge predict modal frequencies and mode shapes that are very close to those identified from ambient vibration measurements.

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